A Method for Incorporating Different Sized Cells into the Finite-Difference Time-Domain Analysis Technique

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***Abstract*— A method is presented that simplifies the process of incorporating a refined mesh around a discontinuity in a finite-difference time-domain algorithm. The technique is used to calculate the tangential electric fields on the boundaries between different sized cells in a variable step size mesh environment. A waveguide is modeled in two dimensions and the accuracy of the mesh refinement algorithm is tested by measuring the amount of field reflected during the transition from a coarse mesh to a fine mesh. The amount of error is found to be less than 1%.**

I. Introduction

Use of the finite-difference time-domain (FDTD) method as a technique for modeling electromagnetic wave propagation was first proposed by Yee [1] in 1966. Since then the FDTD has been successfully used to analyze the properties of both guiding and radiating structures.

One of the dominant aspects governing the accuracy of the FDTD is the size of the spatial increment used in the model. The spatial increment used in the regions that contain sharp discontinuities must be chosen small enough to accurately model the highly nonuniform field distributions. The effect of having a reduced mesh cell size is to increase the computational run time and memory requirements. This can result in the model becoming too large and time consuming to implement. Most structures also contain large regions where the fields vary slowly and smoothly; having a reduced cell size in these areas presents no real advantage. This implies that the application of a localized refined mesh would provide improved accuracy without the resulting considerable increase in the amount of computational storage required.

In the past, there have been a number of different mesh refinement schemes proposed [2], [3]. This letter puts forward a modified form of the variable step size method (VSSM) proposed in 1991 by Zivanovic, Yee, and Mei [4]. The second-order finite-difference equation (SOFDE) is used to calculate the tangential field values at the coarse and refined mesh boundary. It is shown that the modified technique gives a similar level of accuracy to that obtained using the VSSM whilst also providing a number of other advantages.

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II. The Mesh Refinement Technique

When implementing a FDTD algorithm that contains multiple cell sizes, the only field values that cannot be calculated by using the standard FDTD are the tangential electric fields at the interface between the coarse and fine meshes. The fields are calculated using the homogeneous travelling wave equation:

$$\nabla^2 E - \frac{1}{v_p^2}\frac{\partial^2 E}{\partial t^2} = 0. \tag{1}$$

This equation can be approximated by centred differences to form the second-order finite-difference equation (SOFDE). Using Yee's notation, n indicates the time increment; (i, j, k) correspond to the cartesian coordinates (x, y, z); Δx, Δy, and Δz are the spatial increments; and Δt is the corresponding time increment:

$$\begin{aligned} E^{n+1}(i,j,k) &= 2E^n(i,j,k) - E^{n-1}(i,j,k) + v_p^2\Delta t^2 \\ &\times\Bigg[\frac{E^n(i+1,j,k) - 2E^n(i,j,k) + E^n(i-1,j,k)}{\Delta x^2} \\ &+\frac{E^n(i,j+1,k) - 2E^n(i,j,k) + E^n(i,j-1,k)}{\Delta y^2} \\ &+\frac{E^n(i,j,k+1) - 2E^n(i,j,k) + E^n(i,j,k-1)}{\Delta z^2}\Bigg]. \end{aligned} \tag{2}$$

For simplicity, the mesh refinement algorithm will be presented for the two-dimensional case, with $\Delta x = \Delta z$ and assuming TE-wave propagation (E_y, H_x, H_z only). A mesh reduction factor of 4 will be used as depicted in Fig. 1. The time increments used in the FDTD algorithm for the coarse and fine mesh regions will be symbolized by Δt_c and Δt_f respectively, where $\Delta t_c = 4\Delta t_f$. We will only consider the calculation of the E_y field component on the coarse/fine mesh interface, since all other fields can be calculated using the normal FDTD equations.

We begin describing the mesh refinement algorithm (MRA) by assuming a time sequence where $t = n$ represents the present time; all field values, present and past, have been calculated.

Firstly, we calculate the second-order special difference equations, D_i, at each coarse node on the boundary illustrated in Fig. 1. For example, at point 3 this would be

$$D_3 = E_2 + E_4 + E_1 + E_6 - 4E_3. \tag{3}$$

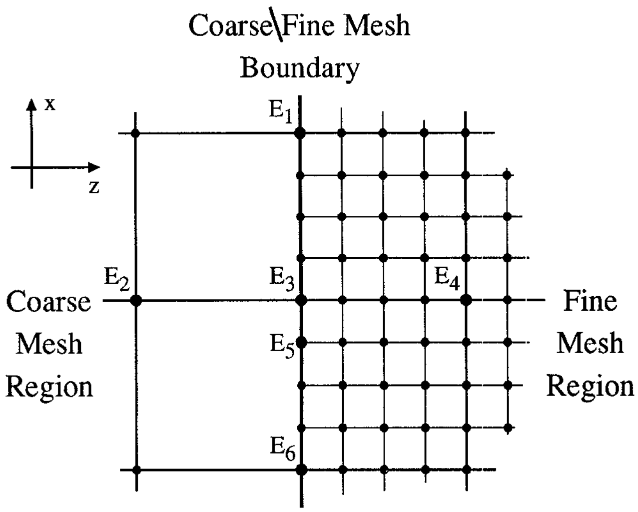

Fig. 1. Region used to calculate the fields at the coarse/fine mesh interface.

Next, the second-order differences are found for the fine mesh nodes by using standard interpolation techniques on the coarse mesh results. For example, at point 5 we would have

$$D_5 = D_3 + \frac{D_6 - D_1}{8} + \frac{D_6 + D_1 - 2D_3}{32}. \quad (4)$$

These differences can now be used to calculate the field values for the next coarse mesh time step and all of the intermediate fine mesh time steps.

At $t = n + \Delta t_f$, all fields on the boundary are calculated using the SOFDE with $\Delta t = \Delta t_f$. The field at point 5 in Fig. 1 would be calculated as

$$E_5^{n+\Delta t_f} = 2E_5^n - E_5^{n-\Delta t_f} + \frac{v_p^2 \Delta t_f^2}{\Delta x^2} D_5. \quad (5)$$

This process is repeated for each time increment up to and including $t = n + \Delta t_c$, after which the whole algorithm is repeated.

The difference between the MRA and the VSSM can be seen in the way that the second-order differences are calculated. For the VSSM, the second-order differences are calculated from spatially interpolated field values, whereas in this letter, the second-order differences are first calculated, then interpolated in space. Reversing the order of the calculations has the advantage of requiring less computational memory and requires less computational time. For a single coarse cell on the boundary, the VSSM requires the memorizing of 15 interpolated field values (assuming a 1:4 reduction), the MRA only requires the memorizing of four second-order differences.

III. Application and Results

The waveguide structure was modeled in two dimensions and excited with a Gaussian pulsed TE_{10} mode. The analysis was carried out twice, firstly with the reduced mesh included half way along the guide, then without. Field values were recorded through time for each to obtain the incident and

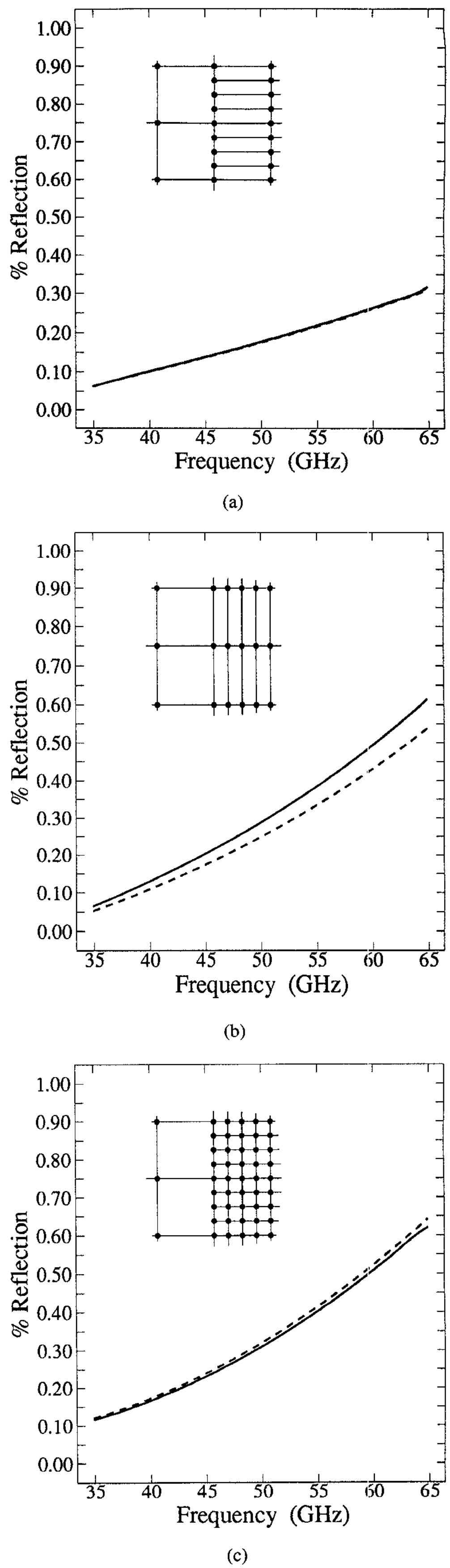

Fig. 2. Reflections from the coarse/fine mesh interface. ——— MRA, - - - - VSSM. (a) transversely subdivided mesh. (b) Longitudinally subdivided mesh. (c) transverse-longitudinally subdivided mesh.

reflected fields. These were Fourier transformed to calculate the frequency-domain reflection coefficient.

Three different arrangements of the fine mesh were investigated. These involved mesh subgridding in the transverse direction, Fig. 2(a), the longitudinal direction, Fig. 2(b), and in both directions, Fig. 2(c). These three different fine mesh cell shapes were used since they represent the different configurations found in FDTD algorithms.

Both the technique outlined in this letter (MRA) and that published by Zivanovic *et al.* [4] (VSSM), were applied to find the percentage reflection from the coarse/fine mesh boundary. The results are demonstrated in Fig. 2(a)–(c) for comparison.

Fig. 2(a) and Fig. 2(c) show that the MRA performs as well as the VSSM for the meshes that are subdivided transversely and transverse-longitudinally. This is because the error produced by the second-order differencing in the modified algorithm is equivalent to that produced by the two-dimensional interpolation in the original VSSM. It can be seen that for both subgridding methods that reflection errors of less than 1% are achievable. In the longitudinally subdivided case, Fig. 2(b), the error is greater since the VSSM algorithm only has a one-dimensional interpolation; fortunately, a mesh of this shape is of little advantage

In a parallel computing environment, the computational speed is governed by the number of serial operations required to carry out a given task. In our analysis, for the transverse-longitudinal case, the MRA required a total of 16 serial calculations less than the VSSM per coarse time step. Our program using the MRA, which was run on a MasPar MP-1 massively parallel computer, was found to run 15% faster than that using the VSSM. In this regard, it can also be seen that the simplicity of the MRA allows for creation of more efficient subdivided mesh algorithms.

IV. Conclusion

An algorithm for incorporating reduced cell sizes into finite-difference time-domain technique has been presented. The accuracy has been shown to be the same as that achieved by previous methods for the majority of applications. The main advantages of this method are that it requires less computational memory and CPU time to apply compared with previous techniques, which are dominant factors governing the use of any finite-difference time-domain analysis. Another positive aspect of this new algorithm is the simplicity by which it can be applied, even when the size of the coarse mesh is not an integer multiple of the fine mesh size.

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